closely with those given by Piers and Bjerram* in various papers in the 'Zeitschrift für Electrochemie' in 1911 and 1912, of which a summary was given by Pye in the 'Automobile Engineer' for February, 1920.

Mr. Womersley states that, at the time the experiments were initiated and carried out, Bjerram's corrections to Pier's results had not arrived; they were to hand before the date of the paper, August, 1921.

A Photographic Spectrum of the Aurora of May 13–15, 1921, *and Laboratory Studies in Connection with it.*

By Lord Rayleigh, F.R.S.

(Received March 6, 1922.)

[Plate 2.]

§ 1. *Aurora of May* 13–15.

On May 13, 1921, and the following days, there was a magnetic storm of almost unprecedented violence, connected without doubt with a large and highly eruptive spot near the centre of the sun's disc.

At the time I was systematically photographing the spectrum of the diffused light of the sky every night for the investigation of the faint aurora line ordinarily present.† The instrument used was the spectrograph No. 1‡ having a Rutherford prism, and a cinematograph lens of 3 inches focus, working at F/1·9. An orthochromatic plate was used. The direction of view was 45° up to the north.

On developing the photograph of May 13 (*i.e.*, begun on the evening of that day) it was found that the nitrogen negative bands had come up strongly, as well as the green aurora line. This was the first intimation I had that anything unusual was in progress, and the day of May 14 was spent in preparing as far as possible for an extended programme if the aurora should continue. Two additional spectrographs were extemporised. The aurora *did* continue on the night of May 14, and presumably throughout the intervening day. Another and somewhat stronger photograph on an orthochromatic plate was got with the Rutherford prism instrument used before.

* Such small differences as there are appear to be due to the fact that Bjerram used Holborn's experimental results. Table I has been calculated from his smoothed curve.

† 'Roy. Soc. Proc.,' A, vol. 100, p. 367 (1921).

‡ *Loc. cit.*, p. 368.

A-three prism spectrograph with another F/1·9 lens showed that the aurora line was quite out of coincidence with the krypton line used for comparison, but unfortunately too long a slit was used, and the lines were too much curved for satisfactory measurement of the interval. For the rest, the three-prism instrument gave a much weaker spectrum than the other, and nothing was gained by using it.

The third instrument had a portrait lens of much smaller relative aperture, and was pointed at the sky 45° up from a southern window. An Ilford panchromatic plate was used, in the hope of getting the red part of the aurora spectrum. Nothing came out except the strongest negative band of nitrogen.

On the night of May 14 the aurora was watched visually. The moon was 7 days' old and set at 1.33 in the morning of May 15. After the moon had set, the aurora was observed high up in the north, with a variety of colours, including shades of purple and violet. It could be seen red through an orange gelatine filter, transmitting from about λ 5800. The clouds interfered considerably, no part of the sky being really clear. The form of the aurora could not be made out.

An orthochromatic plate was exposed to the sky under yellow and blue filters which were of such intensity as to give about equal actinic effects with direct moonlight, or with the light of the sky on ordinary moonless nights.* On this occasion the actinic effect under the blue filter corresponded to a more than tenfold relative intensity. Thus the auroral sky on this occasion was much richer in blue and violet rays than the ordinary night sky. The colour index was more than 2·5 units lower.

The best spectrum was that of May 14, 1921, reproduced in Plate 2, No. 1. The comparison spectrum is a krypton vacuum tube. The spectrum of May 13 was generally very similar, though not quite so intense.

The Table on p. 116 is a list of the lines and bands on the photograph, with intensities, on a scale of ten.

The whole spectrum, then, is accounted for by the chief aurora line of unknown origin, and the negative bands of nitrogen. There is nothing else on the plates except a trace of continuous spectrum, probably due to moonlight.

These plates show considerably more detail in the nitrogen bands than previous photographs. The best previous photograph, so far as I know, is that of Vegard, reproduced in 'Phys. Zeit.,' XIV, p. 680, 1913. This was taken at Bossekop, in the north of Norway, and very long exposures were given. His work had the great merit of proving beyond all doubt that the bands were really negative nitrogen bands.

* See 'Roy. Soc. Proc.,' A, vol. 99, p. 10 (1921).

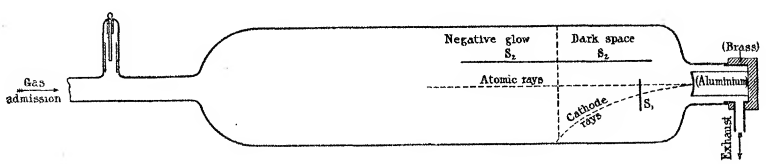
		Intensity.	
		May 13.	May 14.
Green aurora line	5578	8	7
Negative nitrogen bands—			
Blue group	I 4709	3	2
	II 4651	3	3
	III 4599	—	2
	IV 4554	—	1 ?
Violet group	I 4278	9	9
	II 4236	5	5
	III 4199	2	4
	IV 4166	1	1 ?
Ultra-violet group	I 3914	10	10

It is remarkable that the strongest nitrogen bands are much more intense than the green aurora line, though the plates used are very sensitive in the region of the latter.

§ 2. *Laboratory Studies on this Spectrum.*

The opportunities of getting as good a photograph of the auroral spectrum as this one are few, and I was anxious to make the best use of it in determining the conditions for an artificial reproduction of the nitrogen bands exactly as they appear in the aurora. I wished, if possible, to produce them without any admixture of the various other spectra of nitrogen, but have not been wholly successful in doing so, after many attempts. Some relevant facts have, however, been gathered which are not to be found in the published literature.

A discharge tube was made as in the figure, which is one quarter size. The



concave cathode was of aluminium, 1·2 cm. diameter, and was mounted, as shown, in a brass cap cemented with sealing-wax on to the neck of the vessel. A brass tube from the cap leads to the pump. This device was first used by Aston,* and has the great advantage of preventing a parasitic discharge from striking down to the pump through the connecting tubes. In my case it was

* 'Phil. Mag.,' vol. 39, p. 613 (1920).

not necessary to perforate the cathode, and the currents used were not heavy enough to demand water cooling. The anode was also arranged in the way used by Aston (originally by Lodge), with an insulated metal tube round it, so as to act as its own rectifier. For the rest, the figure will explain itself. A stream of rarefied air or nitrogen from a capillary inlet was passed through the tube and the rate of flow controlled by reducing the pressure in the feed reservoir. In this way the gas in the tube was kept pure and the pressure controlled. At low pressures the various features of the cathode discharge can be very well isolated in a large tube of this kind, and the spectra examined separately.

In small discharge tubes it is difficult to trace the boundary between the negative glow and dark space when the latter is much more than about 3 cm. long. The difficulty is mitigated by the use of a large tube, and still more by photography, using a filter of Wood's ultra-violet glass, which cuts off wave-lengths more than about λ 4000. In this way the green luminescence of the glass, which hinders visual observation, is entirely got rid of. Such a photograph of the tube is reproduced in Plate 2, No. 7, scale natural size. The dark space is 5·6 cm. long, and the alternative spark gap between the balls about 1·3 cm.

In this photograph a weak transverse magnetic field deflects the cathode rays to the left, where they were received on a piece of mica, to avoid fracture of the glass. A pencil of undeflected rays proceeds along the axis of the tube. These are the "retrograde rays" examined by Sir J. J. Thomson.* He found hydrogen atoms and molecules and oxygen atoms; also negatively electrified and unelectrified atoms.

The retrograde rays in nitrogen do not seem to have been examined in detail. It will suffice to call them atomic rays in distinction from cathode (electronic) rays.

It is noteworthy that the thin pencil of cathode rays is of very small section compared with the negative glow, and that the track of these rays is as luminous in the dark space as in the negative glow, except for the superposition of the negative glow upon them. The view sometimes expressed or implied that the negative glow is simply an excitation of the gas by the pencil of cathode rays from the cathode, is seen more clearly than usual to be untenable.

The part of the discharge tube under investigation was focussed on the slit of the spectrograph, the same instrument with which the aurora was taken, but not in exactly the same adjustment as before.† The spectrum in Plate 2, No. 2, shows the spectrum of the dark space, which is, of course, only com-

* 'Phil. Mag.,' vol. 24, p. 209 (1912).

† It was disturbed by an accident.

paratively dark, as a background. Crossing it are the narrower spectra of the atomic rays and cathode rays, which are separated by the magnet, as in Plate 2, fig. 7, S_1 in the figure represents diagrammatically the position of the slit, or rather of its image.

In judging of the spectra excited by these rays, it is necessary to allow for and subtract, so to speak, the background spectrum of the dark space.

It is found that the relative intensities within the blue group of negative bands, or within the violet group, vary under different conditions of excitation. This is most conspicuous in the blue group, where the second band may be stronger than the first. In the violet group the first band is always much the strongest. Accordingly, attention is concentrated on the blue group, the components of which are called I, II, III, in order of increasing refrangibility.

In the auroral spectrum of May 13 II is as strong as I. On May 14 II was stronger than I, and III was equal to I.

Let us compare this with the artificial spectra. In the dark space I is much stronger than II and III is barely visible. Where the track of the cathode rays crosses the spectrum, the bands are strengthened, but their relative intensity is not conspicuously changed. Where the atomic rays cross the spectrum II is much stronger, and III is rather stronger than I. Thus the atomic rays give approximately the intensity ratio which obtained in the aurora of May 14. They go slightly beyond the mark, as regards the increased relative intensity of II.

On the other hand, the dark space and the cathode rays showed no approximation to auroral conditions.

To examine the negative glow, the slit was placed parallel to the length of the discharge with its image as at $S_2 S_2$ in the figure, so that half its length was in the dark space, and half in the negative glow (Plate 2, No. 3). The relative intensities of I, II, III, are about the same in the negative glow as in the dark space and in the cathode rays. The negative glow is thus unlike the aurora in this regard.

There are other points of interest brought out by the photograph (Plate 2, No. 3). The bands of the second positive group of nitrogen (see particularly those of wave-lengths 4059, 3998, 3805, 3754, which are marked with dots above the photograph) are much more strengthened in the negative glow than on the negative bands. This behaviour of the second positive group has already been described by Seeliger and Pommeranig.* On the other hand, the aluminium lines 3944 and 3962 (dotted below the photo-

* 'Ann. d. Phys.,' vol. 59, p. 595 (1919).

graph) are stronger in the dark space, which as regards these would be more correctly called a bright space.*

There is a good deal of evidence from the work of Birkeland, Stormer, and Vegard, for attributing the aurora to cathode rays from the sun. The observations which have been described do not seem very easily conformable to this view. It was thought likely that in the blue group of negative bands, II would gain intensity relative to I as the cathode rays became harder. I have not, however, succeeded in tracing such a tendency. A series of exposures were made at decreasing pressures, then with increasing hardness of the cathode rays, the alternative spark between balls in air varying from 0·5 mm. to 17 mm.

At 0·5 mm. alternative spark the cathode rays could not be satisfactorily observed inside the comparatively narrow dark space, as there was not room to separate them from the atomic rays. They had to be observed in the negative glow where they cross one another at the magnetically displaced focus of the concave cathode. But they are so bright at these high pressures that no ambiguity arose from this. As the alternative spark increased the dark space widened, and the cathode ray spectrum could be photographed inside it. Throughout the range from 0·5 mm. to 17 mm. the band I was much stronger than II, and no definite tendency to equalisation was noticed. At 17 mm. alternative spark it was necessary to give 5 hours' exposure, even with the great light grasp of the instruments used, and at still lower pressures the cathode beam faded into invisibility, although the atomic beam could still be traced.

Reduction of pressure has a two-fold effect. It increases the hardness of the cathode rays, and at the same time diminishes the concentration of molecules in their path. In order to examine the luminosity produced by harder rays than this, it would be necessary to control the density of the atmosphere traversed independently. This could, perhaps, be successfully done by the use of Lenard's aluminium window. It is true that by using a denser atmosphere, we should violate the conditions which obtain in the upper air. In observing the aurora we look through immense thicknesses of very feebly luminous gas. In a laboratory imitation these great thicknesses are prohibitive, and it is necessary to increase the luminosity per unit volume of the gas in some way. It may be argued that this changes the spectrum of the light, and spoils comparison with the aurora. Without a full knowledge of the mechanism of luminosity such as we do not possess at present, the criticism cannot altogether be met.

* Possibly these lines should rather be referred to the first negative layer, the limits of which are not easily defined.

So far, then, as the detailed development of the negative bands is concerned, the cathode rays do not imitate the aurora so well as the atomic rays.

The negative bands can also be observed in the "first negative layer," the luminous layer nearest the cathode, which is regarded as continuous with the canal rays, and would be prolonged into the latter if the cathode were perforated. Here, too, the negative bands have the auroral character, II, in the blue group, being equal in intensity to I (Plate 2, No. 4).

Finally, it is possible to observe the negative bands at low pressure in the capillary of a spectrum tube of the ordinary form, thus, in the positive column of the discharge. This observation is due to Hasselberg,* and was used in his measurements of the negative bands. In this case, too, the intensity distribution is like that in the aurora, the development of the later members in each group being exceptionally good. Thus, Hasselberg gives the wave-lengths of six bands in the blue group.† These bands are more like the auroral ones than any others that I have been able to produce artificially, and are shown in comparison with them in Plate 2, No. 5.

If these bands could only be obtained alone, without the second positive group, the comparison would be very satisfactory.

In the capillary at low pressures we have a procession of ions of both signs, but of opposite charges, moving in a strong electrical field.

It appears, then, that an intensity distribution like that in the aurora has not been obtained, except in cases when we can suppose the bombarding particles to be, in part at least, of atomic dimensions. Purely electronic bombardment has not been observed to give it.

This, as far as it goes, would be in favour of the idea which has sometimes been advocated, that rays from the sun of an atomic nature give rise to the aurora. I have not succeeded, however, in getting so simple a spectrum as that of Plate 2, No. 1, in any experimental bombardment of the gas with atomic rays. Let us take, for instance, the spectrum of the first negative layer. I have carefully followed the changes in the spectrum with increasing pressure.

In no case did the negative series of nitrogen bands appear without contamination with other nitrogen spectra. At the lower pressures, the nitrogen *line* 3995 was conspicuous, and of intensity comparable with the blue negative bands I and II. As the pressure increased, this line weakened, and the nitrogen bands of the second positive group, 4059, 3998,

* 'Acad. Imp. des Sciences de St. Petersbourg,' 7th Series, vol. 32, No. 15 (1885).

† It has been suggested that the appearance of negative bands in the spectrum of the capillary is due merely to the ordinary expansion of the negative glow at low pressures. But the same effect is obtained in a specially shaped tube, in which the cathode (and anode) are both "round the corner."

3805, and 3755 came in,* but before the line had faded out. Thus, at no stage is nitrogen represented by negative bands only, as in the aurora spectrum (Plate 2, No. 1).

The best approximation to this state of things was at a dark space of 2 cm. (equivalent spark gap in air, 3 mm.). The spectrum in this case is reproduced, (Plate 2, No. 4), and shows the line 3995 marked by dots, somewhat less intense than the blue negative bands of nitrogen. The pressure is just short of that required to make the positive bands come in. I speak of the pressure because it is the independent variable which is directly observed and controlled; but, of course, indirect and perhaps complex effects of a change of pressure are really concerned.

Again, in no case are the hydrogen lines absent from the experimental spectra. The air, or nitrogen, fed into the tube was dried with phosphorus pentoxide, though I did not specially study the effect of prolonged drying. The hydrogen appears to come from the electrode, the intensity being greatest when the electrode is new, and depending also on the material of which the latter is made. I have tried aluminium, iron, tungsten, and silicon, and experience has been in favour of the latter, which gives a minimum of hydrogen, and, moreover, gives no lines of its own. But even prolonged running of a silicon cathode does not get rid of the hydrogen lines from the first negative layer (Plate 2, No. 4).

Spectrum of the Aurora, and Composition of the Upper Atmosphere.

We have reason to believe, not indeed from direct experiment, but on very strong theoretical grounds, that the upper atmosphere in which the aurora occurs, is rich in the lightest constituents of air, helium and (possibly) hydrogen. The subject is fully discussed in a recent paper by Chapman and Milne.† Do these constituents appear in the auroral spectrum? The photographs reproduced confirms clearly all previous evidence that they do not. We are faced, therefore, with the alternatives that either they are not there, or that the conditions of excitation are not such as to develop the spectrum.

In the case of hydrogen we have no cogent evidence that it is a normal constituent of the lower air, and we may escape from the need for explaining its absence from the aurora on this ground. On any other view a formidable difficulty would be encountered. For, according to all laboratory experience,

* 3998 cannot be well separated from the *line* 3995 with the resolving power used, but the presence or absence of the other second positive bands mentioned serves to discriminate.

† 'J. R. Met. Soc.,' vol. 46, p. 357 (1920).

even a trace of hydrogen asserts itself in the spectrum of any kind of electric discharge through air, under any conditions of pressure.

We may turn to the question of helium, and ask whether, if present in the quantities anticipated from the theory of diffusion, it could fail to be excited by cathode ray or atomic ray bombardment of the gas. It is impossible at present to predict *a priori* the relative visibility of different gases in a mixture; indeed it depends very much on the discharge conditions. It is known that helium may be largely masked by admixture of nitrogen, but this action is most conspicuous in the positive column. To test the visibility of helium in a mixture with nitrogen, five parts of helium with one of nitrogen was fed in through a capillary tube to the large discharge vessel (figure), and the cathode ray and atomic ray spectra photographed at low pressures, the alternative spark gap between balls being held at 1 cm.

The violet and ultra-violet negative nitrogen bands are still the most conspicuous feature in both spectra (Plate 2, No. 6), but the helium lines 4471 and D_3 marked with dots above are comparable in intensity with the blue nitrogen group. In the cathode rays, these lines are scarcely, if at all, visible. On the other hand the green helium line 5016 (marked with a dot below the photograph), is weak in the atomic rays and stronger in the cathode rays, though here only about equal to the second member of the negative nitrogen group, which is comparatively weak in the cathode ray spectrum.

If diffusion in the atmosphere is considered to begin at 20 kilom. up, this composition of five volumes helium to one of nitrogen should prevail at about 130 kilom. But the particular aurora in question extended to far greater altitudes than this. Stormer* found for one long auroral ray heights ranging from 192 to about 470 kilom., so that the whole of this ray (the only feature about which we have published information) lay in an atmosphere which should consist almost entirely of helium. These observations were taken in the neighbourhood of Christiania. But it seems not unreasonable, considering the world-wide distribution of this aurora, to assume that it occurred at about the same height over the south of England.

We may possibly escape from the conclusion that the luminosity photographed in the spectrum (Plate 2, No. 1), occurred in an atmosphere consisting largely of helium, by putting its height near the lower limit of auroral heights (90 kilom.) and by putting the height at which separation of the gases by diffusion begins rather higher than the probable value of 20 kilom. But there is a minimum of room for escaping in this way.

* 'Comptes Rendus,' vol. 172, p. 1672 (1921).

Upon the whole, it appears very difficult on the hypothesis of atomic rays to explain the absence of helium lines from the spectrum and, it may be added, the absence of nitrogen line spectrum.

On the hypothesis of cathode ray excitation, this difficulty would be considerably relieved, but in that case we encounter another difficulty, namely, that the nitrogen band spectrum in the aurora has the intensity distribution characteristic of atomic rays, not of cathode rays.

It seems best at present not to lose sight of the possibility that the mode of excitation may be something entirely different from either.

§ 3. *Summary.*

A photographed spectrum of the aurora obtained on the night of March 14, 1922, is reproduced. It shows the negative bands of nitrogen in considerable detail, also the green aurora line of unknown origin, which, however, is subordinate.

A number of laboratory photographs of the negative bands of nitrogen excited in various ways are reproduced for comparison.

With atomic ray excitation, and still better in the narrow positive column (capillary tube) at low pressure the development of the negative bands can be imitated. But other nitrogen spectra (line spectrum and second positive band spectrum) persistently appear in addition.

The cathode ray spectrum is free from these foreign spectra. But the negative bands produced in this way are not developed like those in the aurora, the intensity being much more concentrated in the first band of each group. Hard and soft cathode rays behave alike in this respect.

The auroral spectrum is also discussed in relation to the upper atmosphere. Assuming that helium is the main constituent above 130 kilom., as the theory of diffusion indicates, then it is difficult, on the hypothesis of positive ray excitation, to explain its absence from the spectrum of this particular aurora, which at Christiania, reached to 470 kilom. Experiments on artificial mixtures indicate that helium should be visible.

With cathode ray excitation, this difficulty would be lessened, but the different development of the nitrogen bands remains.

It is possible that the true mode of excitation of the aurora has not yet been suggested.

DESCRIPTION OF PLATE.

No. 1.—Aurora spectrum with krypton comparison. The yellow-green aurora line, λ 5578, is seen on the right in approximate coincidence with the strong krypton line.

No. 2.—Spectrum of nitrogen, in the cathode rays and atomic rays separately, seen on a background of dark space, which is only relatively dark. Note the development of the various members of the blue group in each case. The atomic rays give an intensity distribution more like the aurora.

No. 3.—Spectrum of negative glow above (second positive group of bands marked) and of dark space below (aluminium lines marked).

No. 4.—Spectrum of first negative layer. Nitrogen line marked.

No. 5.—Spectrum of the capillary of a nitrogen spectrum tube at low pressure, placed in comparison with aurora. Note similar development of bands.

No. 6.—Cathode rays and positive rays in a mixture of five parts helium and one part nitrogen. Helium lines marked.

No. 7.—Direct photograph of the discharge tube as used in obtaining spectra 2, 3, and 6. The dark space traversed by atomic rays (straight) and cathode rays (deflected). Below is the negative glow, then the positive column. The tube is photographed through ultra-violet glass. Scale about one-quarter.

Fourier's Series and Analytic Functions.

By T. CARLEMAN, Docent in Mathematics in the University of Upsala, and G. H. HARDY, F.R.S.

(Received February 2, 1922.)

1. *Introduction.*

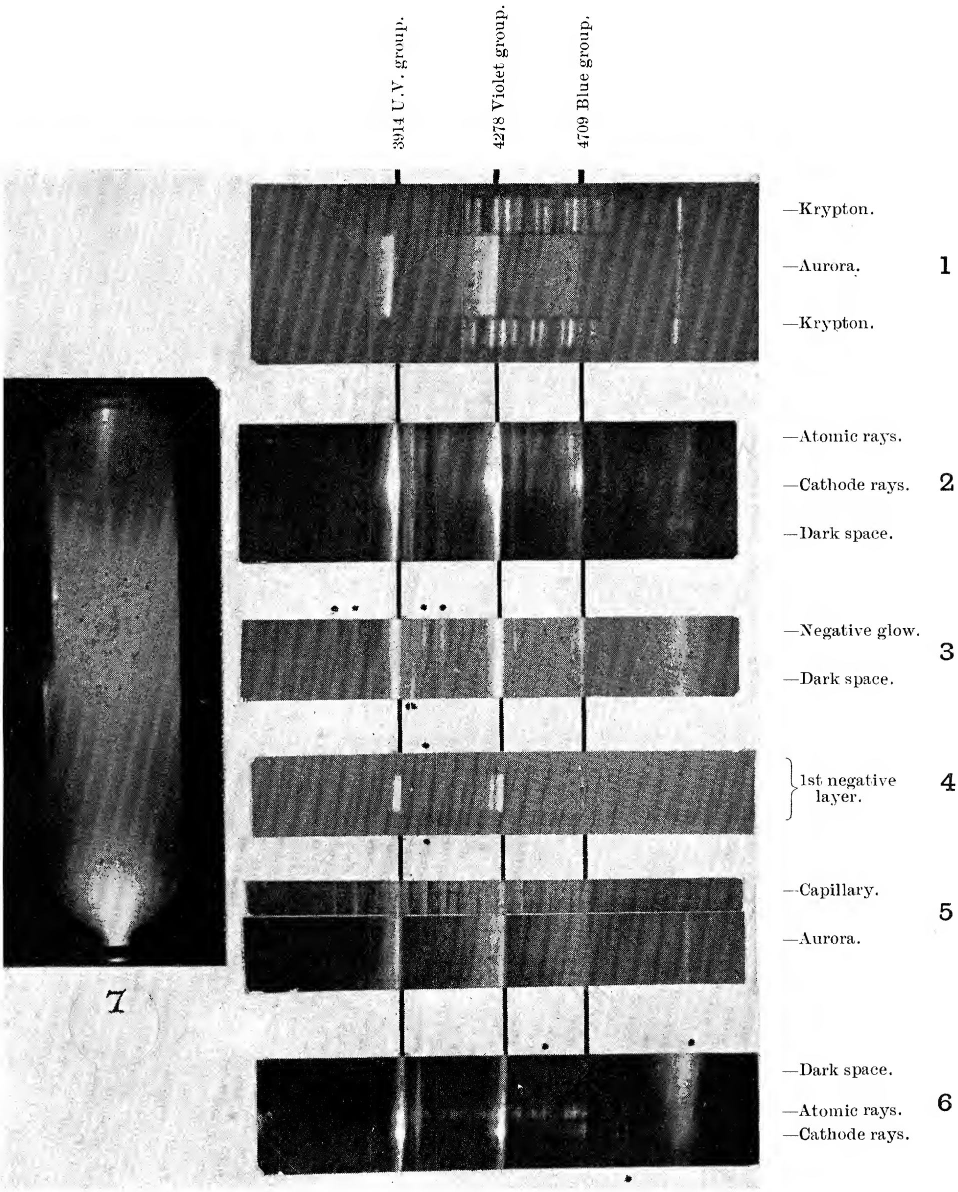
1.1. In the theorems which follow we are concerned with functions $f(x)$ real for real x and integrable in the sense of Lebesgue. We do not, however, remain in the field of the real variable, for we suppose, in §§ 4 *et seq.*, that $f(x)$, or a function associated with $f(x)$, is analytic, or, at any rate, harmonic, in a region of the complex plane associated with the particular real value of x considered.

The Fourier's series considered are those associated with the interval $(0, 2\pi)$. If a is a point of the interval, we write

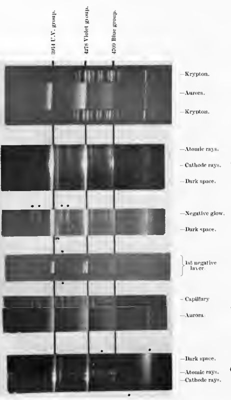
$$\phi(u) = \tfrac{1}{2}\{f(a+u)+f(a-u)-2s\} \qquad (0 < a < 2\pi), \tag{1.11}$$

$$\phi(u) = \tfrac{1}{2}\{f(u)+f(2\pi-u)-2s\} \qquad (a = 0,\ a = 2\pi), \tag{1.12}$$

where s is a constant.


3914 U.V. group.
4278 Violet group.
4709 Blue group.
—Krypton.
—Aurora.
—Krypton.
1
—Atomic rays.
—Cathode rays.
—Dark space.
2
—Negative glow.
—Dark space.
3
1st negative layer.
4
—Capillary.
—Aurora.
5
—Dark space.
—Atomic rays.
—Cathode rays.
6
7

7



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